On Petri-net synthesis and attribute-based visualization

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Abstract. State space visualization is important for a good understanding of the system's behavior. Unfortunately, today's visualization tools typically ignore the fact that states might have attributes. Based on these attributes, some states can be considered equivalent after abstraction, and can thus be clustered, which simplifies the state space. *Attribute-based visualization tools* are the exception to this rule. These tools can deal with attributes. In this paper, we investigate an approach based on *Petri nets*. Places in a Petri net correspond in a straightforward way to attributes. Furthermore, we can use existing techniques to automatically derive a Petri net from some state space, that is, to automatically add attributes to that state space. As a result, we can use attribute-based visualization tools for any state space. Unfortunately, the approach is hampered by the fact that not every state space results in a usable Petri net.

1 Introduction

State spaces are popular for the representation and verification of complex systems [4]. System behavior is modeled as a number of *states* that evolve over time by following *transitions*. Transitions are "source-action-target" triplets where the execution of an *action* triggers a change of state. By analyzing state spaces more insights can be gained into the systems they describe.

In this paper, we assume the presence of state spaces that are obtained via *model-based state space generation* or through *process mining* [1]. Given a process model expressed in some language with formal semantics (e.g., Petri nets, process algebras, state charts, EPCs, UML-ADs, MSCs, BPEL, YAWL, etc.), it is possible to construct a state space (assuming it is finite). Often the operational semantics of these languages are given in terms of transition systems, making the state space generation trivial.

State spaces describe system behavior at a low level of detail. A popular analysis approach is to specify and check requirements by inspecting the state space, e.g., model checking approaches [7]. For this approach to be successful, the premise is that all requirements are known. When this is not the case, the system cannot be verified.

Interactive visualization is another technique for studying state spaces. We argue that it offers three advantages:

1. By giving visual form to an abstract notion, communication among analysts and with other stakeholders is enhanced.
2. Users often do not have precise questions about the systems they study, they simply want to "get a feeling" for their behavior. Visualization allows them to start formulating hypotheses about system behavior.
3. Interactivity provides the user with a mechanism for analyzing particular features and for answering questions about state spaces and the behavior they describe.

Attribute-based visualization enables users to analyze state spaces in terms of attributes associated with every state. Users typically understand the meaning of this data and can use this as a starting point for gaining further insights. For example, by clustering on certain data, the user can obtain an abstract view (details on the non-clustered data have been left out) on the entire state space. Based on such a view, the user can come to understand how the originating system behaves with respect to the clustered data. In this paper we investigate the possibility of automatically deriving attribute information for visualization purposes. To do so, we use existing synthesis techniques to generate a Petri net from a given state space. The places of this Petri net are considered as new derived state attributes.

The remainder of the paper is structured as follows. Section 2 provides a concise overview of Petri nets, the *Petrify* tool, and the *DiaGraphica* tool. The *Petrify* tool implements the state-of-the-art techniques to derive a Petri net from a state space, whereas *DiaGraphica* is a state-of-the-art attribute-based visualization tool. Section 3 discusses the approach using both *Petrify* and *DiaGraphica*. Section 4 shows, using a small example, how the approach could work, whereas Section 5 discusses the challenges we faced while using the approach. Finally, Section 6 concludes the paper.

2 Preliminaries

2.1 Petri nets

A classical Petri net can represented as a triplet (P, T, F) where P is the set of places, T is the set of Petri net transitions[1], and $F \subseteq (P \times T) \cup (T \times P)$ the set of arcs. For the state of a Petri net only the set of places P is relevant, because the network structure of a Petri net does not change and only the distribution of tokens over places changes. A state, also referred to as *marking*, corresponds to a mapping from places to natural numbers. Any state s can be presented as $s \in P \rightarrow \{0, 1, 2, \ldots\}$, i.e., a state can be considered as a multiset, function, or vector. The combination of a Petri net (P, T, F) and an initial state s is called

[1] The transitions in a Petri net should not be confused with transitions in a state space, i.e., one Petri net transition may correspond to many transitions in the corresponding state space. For example, many transitions in Fig. 2 refer to the Petri net transition $t1$ in Fig. 3.

a marked Petri net (P, T, F, s). In the context of state spaces, we use places as attributes. In any state the value of each place attribute is known: $s(p)$ is the value of attribute $p \in P$ in state s.

A Petri net also comes with an unambiguous visualization. Places are represented by circles or ovals, transitions by squares or rectangles, and arcs by lines. Using existing layout algorithms, it is straightforward to generate a diagram for this, for example, using *dot* [10].

2.2 Petrify

The *Petrify* [6] tool is based on the Theory of Regions [9, 11, 5]. Using regions it is possible to synthesize a finite transition system (i.e., a state space) into a Petri net.

A (labeled) transition system is a tuple $TS = (S, E, T, s_i)$ where S is the set of states, E is the set of events, $T \subseteq S \times E \times S$ is the transition relation, and $s_i \in S$ is the initial state. Given a transition system $TS = (S, E, T, s_i)$, a subset of states $S' \subseteq S$ is a *region* if for all events $e \in E$ one of the following properties holds:

– all transitions with event e *enter the region*, i.e., for all $s_1, s_2 \in S$ and $(s_1, e, s_2) \in T$: $s_1 \notin S'$ and $s_2 \in S'$,
– all transitions with event e *exit the region*, i.e., for all $s_1, s_2 \in S$ and $(s_1, e, s_2) \in T$: $s_1 \in S'$ and $s_2 \notin S'$, or
– all transitions with event e *do not "cross" the region*, i.e., for all $s_1, s_2 \in S$ and $(s_1, e, s_2) \in T$: $s_1, s_2 \in S'$ or $s_1, s_2 \notin S'$.

The basic idea of using regions is that each region S' corresponds to a place in the corresponding Petri net and that each event corresponds to a transition in the corresponding Petri net. Given a region all the events that *enter* the region are the transitions producing tokens for this place and all the events that *exit* the region are the transitions consuming tokens from this place. Fig. 1 illustrates how regions translate to places. A region r referring to a set of states in the state space is mapped onto a place: a and b enter the region, c and d exit the region, and e and f do not cross the region.

In the original theory of regions many simplifying assumptions are made, e.g., elementary transitions systems are assumed [9] and in the resulting Petri net there is one transition for each event. Many transition systems do not satisfy such assumptions. Hence many refinements have been developed and implemented in tools like *Petrify* [5, 6]. As a result it is possible to synthesize a suitable Petri net for any transition system. Moreover, tools such as *Petrify* provide different settings to navigate between compactness and readability and one can specify desirable properties of the target model. For example, one can specify that the Petri net should be free-choice. For more information we refer to [5, 6].

With a state space as input *Petrify* derives a Petri net for which the reachability graph is bisimilar to the original state space. We already mentioned that the Petri net shown in Fig. 3 can be synthesized from the state space depicted in

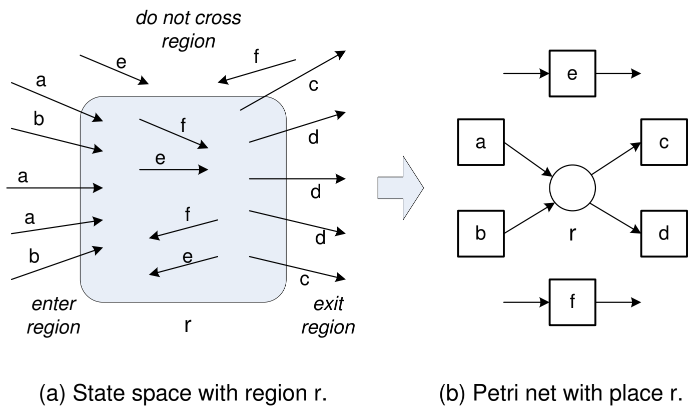


Fig. 1. Translation of regions to places.

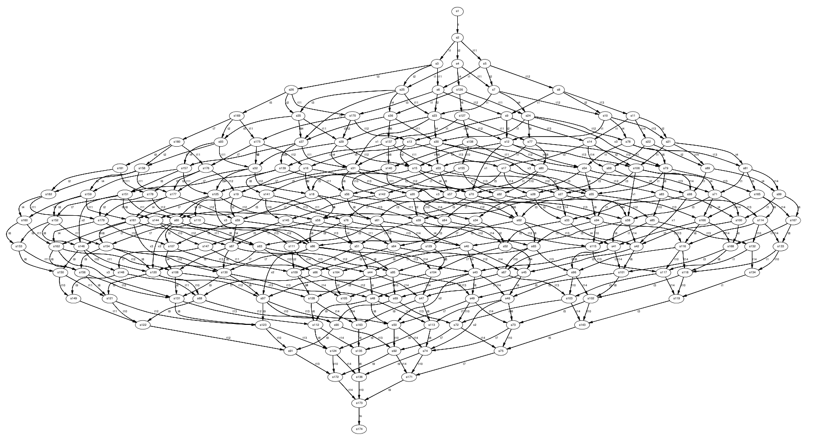


Fig. 2. State space visualization with off-the-shelf graph-drawing tools.

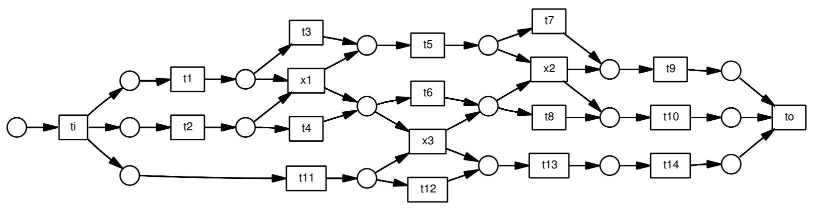


Fig. 3. Petri net synthesized from the state space in Fig. 2.

Fig. 2. This Petri net is indeed bisimilar to the state space. For the sake of completeness we mention that we used *Petrify* version 4.1 (www.lsi.upc.es/petrify/) with the following options: `-d2` (debug level 2), `-opt` (find the best result), `-p`

(generate a pure Petri net), `-dead` (do not check for the existence of deadlock states), and `-ip` (show implicit places).

2.3 DiaGraphica

DiaGraphica is a prototype for the interactive visual analysis of state spaces with attributes and can be downloaded from www.win.tue.nl/˜apretori/diagraphica/. It builds on a previous work [12] and it primary purpose is to address the gap between the semantics that users associate with attributes that describe states and their visual representation. To do so, the user can define custom diagrams that reflect associated semantics. These diagrams are incorporated into a number of correlated visualizations.

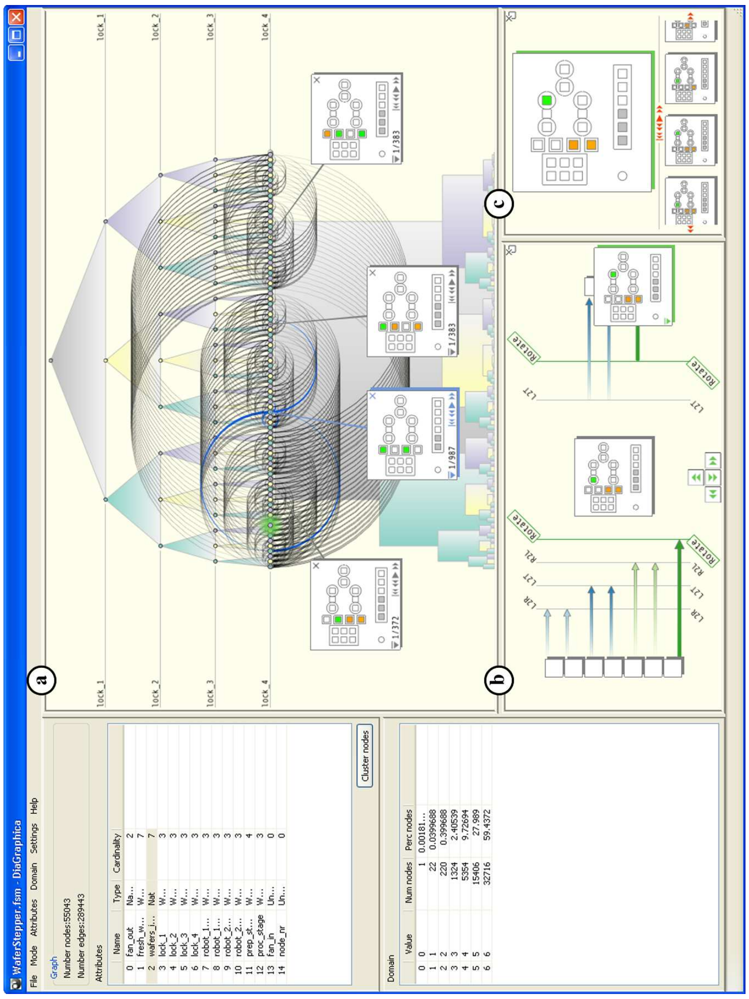
Diagrams are composed of a number of shapes such as ellipses, rectangles and lines. Every shape has a number of Degrees Of Freedom (DOFs) such as position and color. It is possible to define a range of values for a DOF. Such a DOF is then parameterized by linking it with a state attribute. For an attribute-DOF pair the values for the DOF are calculated by considering the values assumed by the attribute.

In the context of this paper this translates to the following. Suppose we have a state space that has been annotated with attributes that describe the different places in its associated Petri net. It is possible to represent this Petri net with a diagram composed out of a number of circles, squares and lines corresponding to its places, transitions and arcs. Now, we can parameterize all circles in this diagram by linking one of more of their DOFs with the attributes representing their corresponding places. For example, their colors can be parameterized such that circles assume a specific color only when the corresponding place is marked.

DiaGraphica has a file format for representing parameterized diagrams. The tool was originally developed with the aim of enabling users to edit and save custom diagrams. However, this facility also makes it possible to import diagrams regardless of where they originate from. Moreover, this allows us to import Petri nets generated with *Petrify* as diagrams.

Parameterized diagrams are used in a number of correlated visualizations. As starting point the user can perform attribute based clustering. The results are visualized in the cluster view (see Fig. 4(a)). Here a node-link diagram, a bar tree and an arc diagram are used to represent the clustering hierarchy, the number of states in every cluster, and the aggregated state space [12]. By clicking on clusters they are annotated with diagrams where the DOFs of shapes are calculated as outlined above. A cluster can contain more than one state and it is possible to step through the associated diagrams and transitions. Transitions are visualized as arcs. The direction of transitions is encoded by the orientation of the arcs which are interpreted clockwise.

The user can also load a diagram into the simulation view as shown in Fig. 4(b). This visualization shows the "current" state as well as all incoming and outgoing states as diagrams. This enables the user to explore a local neighborhood around an area of interest. Transitions are visualized by arrows and an overview of all action labels is provided. The user can navigate through



Graph
Number nodes:55043
Number edges:289443

Attributes

	Name	Type	Cardinality
0	fan_out	Na...	2
1	fresh_w...	W...	7
2	wafers_i...	Nat	7
3	lock_1	W...	3
4	lock_2	W...	3
5	lock_3	W...	3
6	lock_4	W...	3
7	robot_1...	W...	3
8	robot_1...	W...	3
9	robot_2...	W...	3
10	robot_2...	W...	3
11	prep_st...	W...	4
12	proc_stage	W...	3
13	fan_in	Un...	0
14	node_nr	Un...	0

Domain

	Value	Num nodes	Perc nodes
0	0	1	0.00181...
1	1	22	0.0399688
2	2	220	0.399688
3	3	1324	2.40539
4	4	5354	9.72694
5	5	15406	27.989
6	6	32716	59.4372

Fig. 4. DiaGraphica incorporates a number of correlated visualizations that use parameterized diagrams.

the state space by selecting any incoming our outgoing diagram, by using the keyboard or by clicking on navigation icons. Consequently, this diagram slides toward the center and all incoming and outgoing diagrams are updated.

The inspection view enables the user to inspect interesting diagrams more closely and to temporarily store them (see Fig. 4(c)). First, it serves as a magnifying glass. Second, the user can use the temporary storage facility. Users may, for instance, want to keep a history, store a number of diagrams from various locations in the state space to compare, or keep diagrams as seeds for further discussions with colleagues. These are visualized as a list of diagrams through which the user can scroll.

Diagrams can be seamlessly moved between different views by clicking on an icon on the diagram. To maintain context, the current selection in the simulation or inspection view is highlighted in the clustering hierarchy.

3 Using petrify to obtain attributed state spaces

Fig. 5 illustrates the approach. The behavior of systems can be captured in many ways. For instance, as an *event log*, as a formal *model* or as a *state space*. Typically, system behavior is not directly described as a state space. However, as already mentioned in the introduction, it is possible to generate state spaces from process models (i.e., model-based state space generation) and event logs (i.e., process mining [1, 3]). This is shown by the two arrows in the lower left and right of the figure. The arrow in the lower right shows that using model-based state space generation the behavior of a (finite) model can be captured as a state space. The arrow in the lower left shows that using process mining the behavior extracted from an event log can be represented as a state space [2]. Note that an event log provides execution sequences of a (possibly unknown) model. The event log does not show explicit states. However, there are various ways to construct a state representation for each state visited in the execution sequence, e.g., the prefix or postfix of the execution sequence under consideration. Similarly transitions can be distilled from the event log, resulting in a full state space. Fig. 5 also shows that there is a relation between event logs and models, i.e., a model can be used to generate event logs with example behavior and based on an event log there may be process mining techniques to directly extract models, e.g., using the α-algorithm [3] a representative Petri net can be discovered based on an event log with example behavior. Since the focus is on state space visualization, we do not consider the double-headed arrow at the top and focus on the lower half of the diagram.

We make a distinction between event logs, models and state spaces that have *descriptive attributes* and those that do not (inner and outer sectors of Fig. 5). For example, it is possible to model behavior simply in terms of transitions without providing any further information that describes the different states that a system can be in. Fig. 2 shows a state space where nodes and arcs have labels but without any attributes associated to states. In some cases it is possible to attach attributes to states. For example, in a state space generated from a Petri net, the token count for each state can be seen as a state attribute. When a state space is generated using process mining techniques, the state may have state attributes referring to activities or documents recorded earlier.

It is far from trivial to generate state spaces that contain state attributes from event logs or models where this information is absent. Moreover, there may be an abundance of possible attributes making it is difficult to select the attributes relevant for the behavior. For example, a variety of data elements may be associated to a state, most of which do not influence the occurrence of events. Fortunately, as the upward pointing arrow in Fig. 5 shows, tools like *Petrify* can *transform a state space without attributes into a state space with*

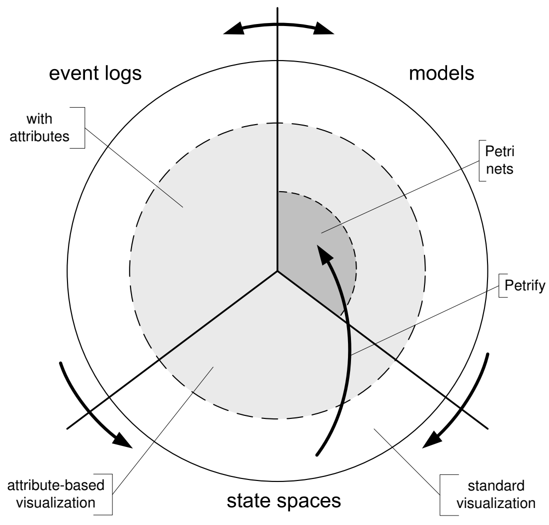


Fig. 5. The approach proposed in this paper.

attributes. Consider the state space in Fig. 2. Since it does not have any state attributes, we cannot employ attribute-based visualization techniques. When we perform synthesis, we derive a Petri net that is guaranteed to be bisimilar to this state space. That is, the behavior described by the Petri net is equivalent to that described by the state space [6]. Fig. 3 shows a Petri net derived using *Petrify*.

Note that the approach, as illustrated in Fig. 5, does not require starting with a state space. Any Petri net can also be handled as input, provided that its state space can be constructed within reasonable time. For a bounded Petri net, this state space is its reachability graph, which will be finite. The approach can also be extended for unbounded nets by using the coverability graph. In this case, $s \in P \rightarrow \{0, 1, 2, \ldots\} \cup \{\omega\}$ where $s(p) = \omega$ denotes that the number of tokens in p is unbounded. This can also be visualized in the Petri net representation. We also argue that our technique is applicable to other graphical modeling languages with some form of semantics, e.g., the various UML diagrams describing behavior. In the context of this paper, we use state spaces as starting point since, in practice, one will encounter a state space more easily than a Petri net.

4 Proof of concept

To illustrate how the approach can assist users, we now present a small case study, using the implementation of the approach as sketched in [14]. Fig. 6 illustrates the route we have taken in terms of the strategy introduced in Section 3. We started with an event log from the widely used workflow management system *Staffware*. The log contained 28 process instances (cases) and 448 events. Fig. 7 shows a fragment of this log after importing it into the *ProM* framework[8, 13].

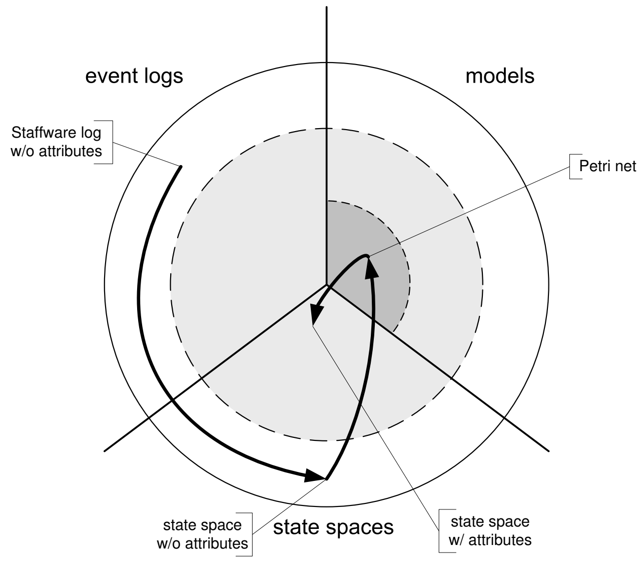
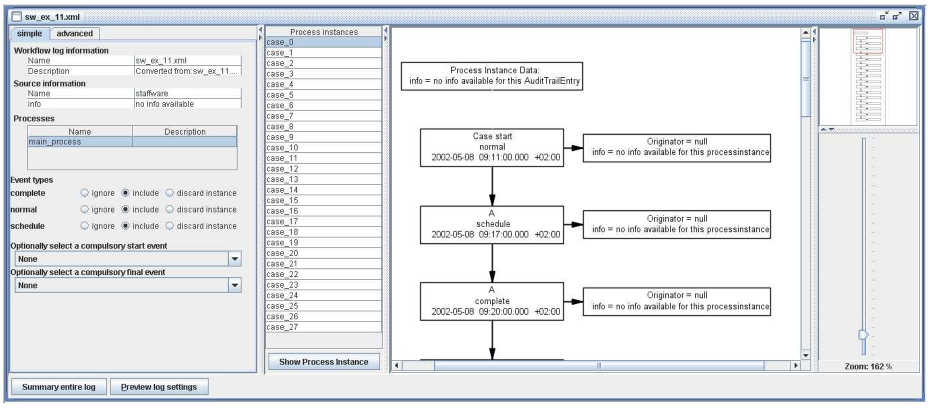

event logs
models
Staffware log w/o attributes
Petri net
state space w/o attributes
state spaces
state space w/ attributes

Fig. 6. The approach taken with the case study.



Workflow log information

Name	sw_ex_11.xml
Description	Converted from:sw_ex_11....

Source information

Name	staffware
info	no info available

Processes

Name	Description
main_process	

Event types

complete: ignore [✓ include] discard instance
normal: ignore [✓ include] discard instance
schedule: ignore [✓ include] discard instance

Optionally select a compulsory start event: None
Optionally select a compulsory final event: None

case_0
case_1
case_2
case_3
case_4
case_5
case_6
case_7
case_8
case_9
case_10
case_11
case_12
case_13
case_14
case_15
case_16
case_17
case_18
case_19
case_20
case_21
case_22
case_23
case_24
case_25
case_26
case_27

Process Instance Data:
info = no info available for this AuditTrailEntry

Case start
normal
2002-05-08 09:11:00.000 +02:00

Originator = null
info = no info available for this processinstance

A
schedule
2002-05-08 09:17:00.000 +02:00

Originator = null
info = no info available for this processinstance

A
complete
2002-05-08 09:20:00.000 +02:00

Originator = null
info = no info available for this processinstance

Fig. 7. A snapshot of *ProM* with the imported *Staffware* log.

Next, we generated a state space from the event log, using the *Transition System Generator* plug-in in *ProM*[2]. The resulting state space is shown in Fig. 8. From the state space, we derived a Petri net using *Petrify* (see Fig. 9). Finally, all states in the state space were annotated with the places of this Petri net as attributes.

It is possible to take different perspectives on the state space by clustering based on different subsets of state attributes. For example, we were interested in

[2] In *ProM* the following options were selected: *Generate TS with Sets (Basic Algorithm)*, *Extend Strategy (Algorithm adds Additional Transitions)*, *The Log has Timestamps*, *Use IDs (numbers) as State Names*, and *Add Explicit End State* [2].

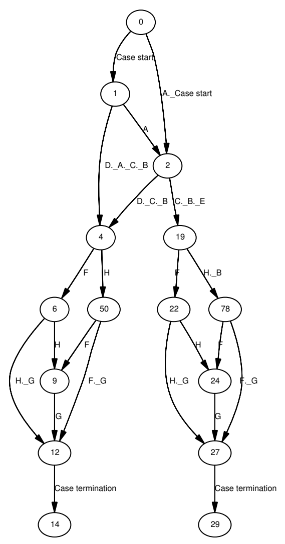


Fig. 8. The state space generated from the *Staffware* log.

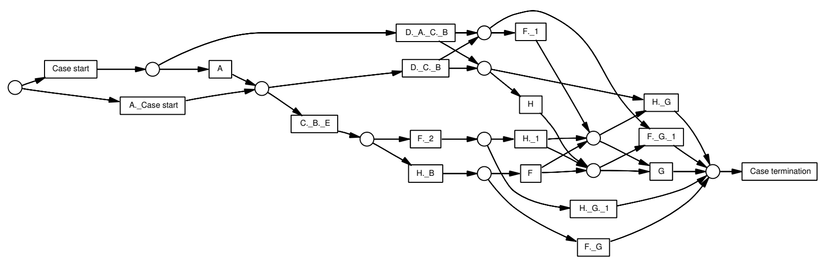


Fig. 9. The Petri net derived from the state space in Fig. 8.

studying it from the perspective of the places `p3`, `p6`, `p7` and `p10` (see Fig. 10). When we clustered the state space based on these places we got the clustering hierarchy shown at the top of Fig. 10.

Next, we clicked on the leaf clusters and considered the marked Petri nets corresponding to these. From these diagrams we learned that `p3`, `p6`, `p7` and `p10` contain either no tokens (Fig. 10(a)) or exactly two of these places contain a token (Fig. 10(b)–(f))[3]. By considering the clustering hierarchy and these diagrams we also discover the following place invariant: `(p3 + p10) = (p6 + p7)`. That is, if `p3` or `p10` are marked, then either `p6` or `p7` is marked and vice versa.

By considering the arcs between the leaf nodes of the clustering hierarchy we learned that there is no *unrelated* behavior possible in the net while one of

[3] The tokens in these places may be hard to see in Fig. 10. However, using the fact that a darker colored node means no tokens whereas a lighter colored node means one token, we can simply derive the markings from the tree. As an example, in the middle diagram (c), place `p3` contains no tokens whereas place `p6` contains one token.

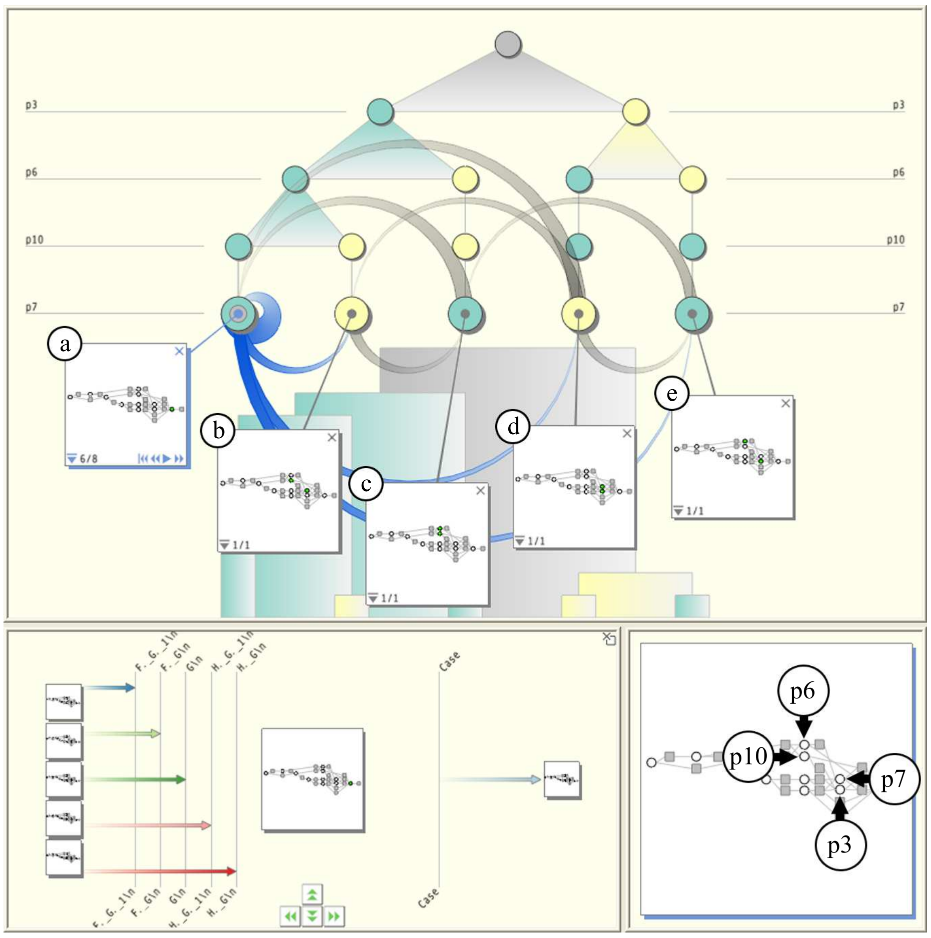


Fig. 10. Visualizing event log behavior using a state space and Petri net.

the places `p3`, `p6`, `p7` or `p10` is marked: every possible behavior changes at least one of these four places. This holds because the only leaf node that contains a self-loop, represented by an arc looping back to its originating cluster, is the left-most cluster. However, as we noted above, this cluster contains all states where neither `p3`, `p6`, `p7` nor `p10` are marked. As an aside, by loading the current state into the simulation view at the bottom right, we saw that this state has five possible predecessors but only a single successor.

We also clustered on all places not selected in the previous clustering. This results in the clustering in Fig. 11. In a sense this can be considered as the dual of the previous clustering. Here we found an interesting result. Note the diagonal line formed by the lighter colored clusters in Fig. 11. Below these clusters the clustering tree does not branch any further. This means that only one of the places, apart from those we considered above (`p3`, `p6`, `p7` and `p10`), can be

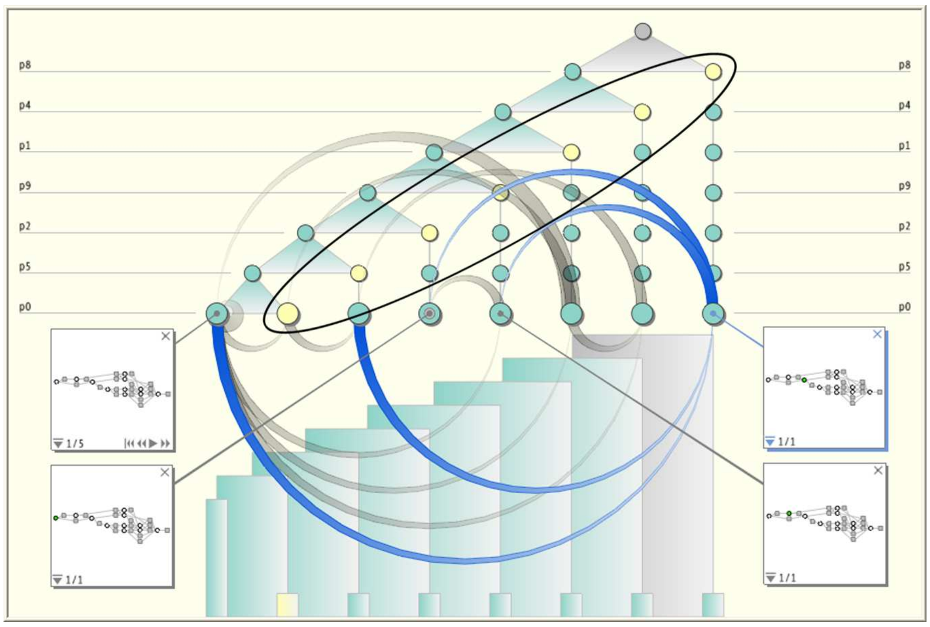


Fig. 11. Visualizing the clustering dual of Fig. 10.

marked at the same time. Again, this observation is much more straightforward when we consider the diagrams representing these clusters. The leaf clusters below the diagonal contain no self loops. Similar to an earlier observation, this means that there is no unrelated behavior possible in the net while any one of the places we consider is marked.

5 Challenges

The synthesis of Petri nets from state spaces is a bottle-neck. More specifically, we have found the space complexity to be an issue when attempting to derive Petri nets from state spaces using *Petrify*. *Petrify* is unable to synthesize Petri nets from large state spaces if there is little "true" concurrency. If a and b can occur in parallel in state s_1, there are transitions $s_1 \xrightarrow{a} s_2$, $s_1 \xrightarrow{b} s_3$, $s_2 \xrightarrow{b} s_4$, and $s_3 \xrightarrow{a} s_4$ forming a so-called "diamond" in the state space. If there are fewer "diamonds" in the state space, this results in a poor confluence and Petri nets with many places.

Fig. 12 shows a Petri net obtained by synthesizing a relatively small state space consisting of 96 nodes and 192 edges. The resulting net consists of 50 places and more than 100 transitions (due to label splitting) and is not very usable and even less readable than the original state space. This is caused by the poor confluence of the state space and the resulting net nicely shows the limitations of applying regions to state spaces with little true concurrency.

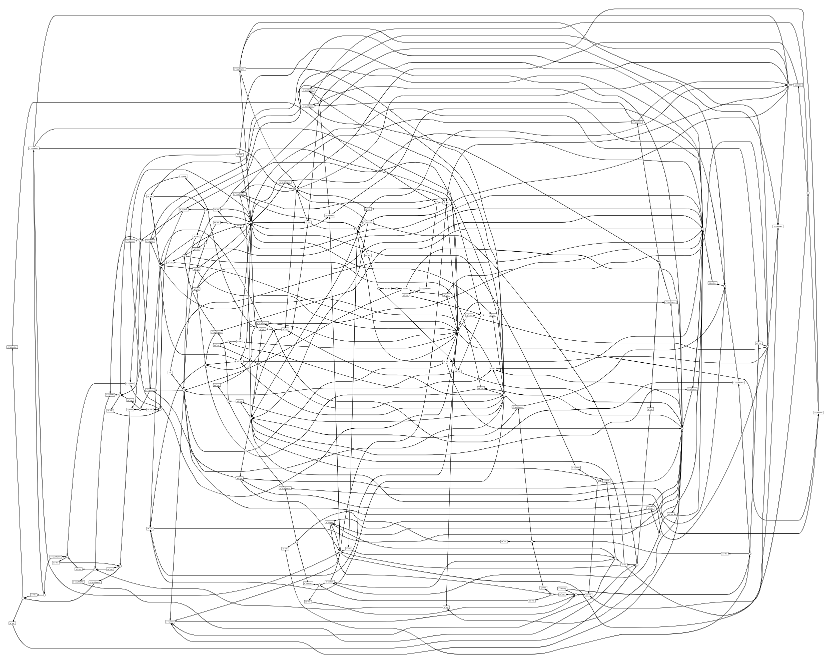


Fig. 12. Suboptimal Petri net derived for a small state space.

Fig. 12 illustrates that the use of *Petrify* is not always suitable. If the system does not allow for a compact and intuitive representation in terms of a labeled Petri net, it is probably *not useful to try and represent the system state in full detail.* Hence more abstract representations are needed when showing the individual states. The abstraction does not need to be a Petri net. However, even in the context of regions and Petri nets, there are several straightforward *abstraction mechanisms.*

First of all, it is possible to split the sets of states and transitions into interesting and less interesting. For example, in the context of process mining states that are rarely visited and/or transitions that are rarely executed can be left out using abstraction or encapsulation. There may be other reasons for removing particular transitions, e.g., the analyst rates them as less interesting. Using abstraction (transitions are hidden, i.e., renamed to τ and removed while preserving branching bisimilarity) or encapsulation (paths containing particular transitions are blocked), the state space is effectively reduced. The reduced state space will be easier to inspect and allows for a simpler Petri net representation.

Another approach is not to simplify the state space but to generate a model that serves as a simplified *over-approximation of the state space.* Consider for example Fig. 12 where the complexity is mainly due to the non-trivial relations between places and transitions. If places are removed from this model, the resulting Petri net is still able to reproduce the original state space (but most likely also allows for more and infinite behavior). In terms of regions this corresponds to only including the most “interesting” regions resulting in an over-approximation of the state space. Future research aims at selecting the right abstractions and over-approximations.

6 Conclusions and future work

In this paper we have investigated an approach for state space visualization with Petri nets. Using existing techniques we derive Petri nets from state spaces in an automated fashion. The places of these Petri are considered as newly derived attributes that describe every state. Consequently, we append all states in the original state space with these attributes. This allows us to apply a visualization technique where attribute-based visualizations of state spaces are annotated with Petri net diagrams.

The approach provides the user with two representations that describe the same behavior: state spaces and Petri nets. These are integrated into a number of correlated visualizations. By presenting a case study, we have shown that the combination of state space visualization and Petri net diagrams assists users in visually analyzing system behavior.

We argue that the combination of the above two visual representations is more effective than any one of them in isolation. For example, using state space visualization it is possible to identify all states that have a specific marking for a subset of Petri net places. Using the Petri net representation the user can consider how other places are marked for this configuration. If we suppose that the user has identified an interesting marking of the Petri net, he or she can identify all its predecessor states, again by using a visualization of the state space. Once these are identified, they are easy to study by considering their Petri net markings.

In this paper, we have taken a step toward state space visualization with automatically generated Petri nets. As we have shown in Section 4, the ability to combine both representations can lead to interesting discoveries. The approach also illustrates the flexibility of parameterized diagrams to visualize state spaces. In particular, we are quite excited about the prospect of annotating visualizations of state spaces with other types of automatically generated diagrams.

Finally, as indicated in Section 5, current synthesis techniques are not always suitable: If not elegant Petri net exists for a given state space, than *Petrify* will not be able to find such a net. In such a situation, allowing for some additional behavior in the Petri net, that is, by over-approximating the state space, might result in a far more elegant net. Therefore, we are interested in automated abstraction techniques and over-approximations of the state space. Of course, there’s also a downside: The state space corresponding to the resulting Petri net is not longer bisimilar to the original state space. Nevertheless, we feel that having an elegant approximation is better than having an exact solution that is of no use.

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